

APPENDIX E |

MATHEMATICAL

FORMULAS

Quadratic formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Triangle of base b and height h	Area $= \frac{1}{2}bh$	
Circle of radius r	Circumference $= 2\pi r$	Area $= \pi r^2$
Sphere of radius r	Surface area $= 4\pi r^2$	Volume $= \frac{4}{3}\pi r^3$
Cylinder of radius r and height h	Area of curved surface $= 2\pi rh$	Volume $= \pi r^2 h$

Table E1 Geometry

Trigonometry

Trigonometric Identities

- $\sin \theta = 1/\csc \theta$
- $\cos \theta = 1/\sec \theta$
- $\tan \theta = 1/\cot \theta$
- $\sin(90^\circ - \theta) = \cos \theta$
- $\cos(90^\circ - \theta) = \sin \theta$
- $\tan(90^\circ - \theta) = \cot \theta$
- $\sin^2 \theta + \cos^2 \theta = 1$
- $\sec^2 \theta - \tan^2 \theta = 1$
- $\tan \theta = \sin \theta / \cos \theta$
- $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
- $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$
- $\sin 2\theta = 2\sin \theta \cos \theta$
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta$

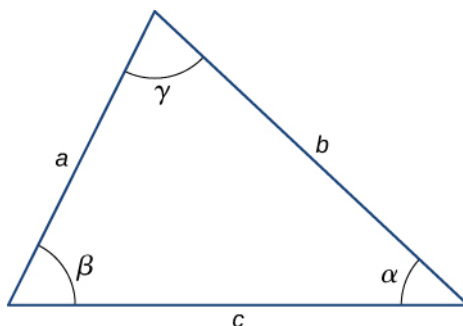
$$15. \sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$16. \cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

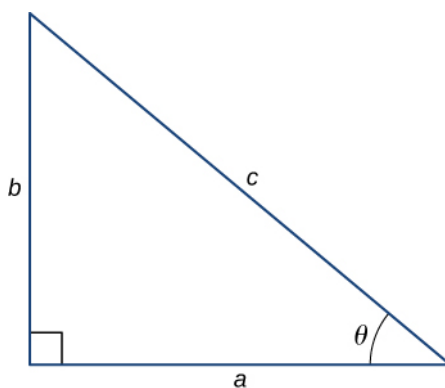
Triangles

$$1. \text{ Law of sines: } \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$2. \text{ Law of cosines: } c^2 = a^2 + b^2 - 2ab \cos \gamma$$



$$3. \text{ Pythagorean theorem: } a^2 + b^2 = c^2$$



Series expansions

$$1. \text{ Binomial theorem: } (a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \dots$$

$$2. (1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \dots (x^2 < 1)$$

$$3. (1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \mp \dots (x^2 < 1)$$

$$4. \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$5. \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$6. \tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$$

$$7. e^x = 1 + x + \frac{x^2}{2!} + \dots$$

$$8. \ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots (|x| < 1)$$

Derivatives

1. $\frac{d}{dx}[af(x)] = a\frac{d}{dx}f(x)$
2. $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
3. $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$
4. $\frac{d}{dx}f(u) = \left[\frac{d}{du}f(u)\right]\frac{du}{dx}$
5. $\frac{d}{dx}x^m = mx^{m-1}$
6. $\frac{d}{dx}\sin x = \cos x$
7. $\frac{d}{dx}\cos x = -\sin x$
8. $\frac{d}{dx}\tan x = \sec^2 x$
9. $\frac{d}{dx}\cot x = -\csc^2 x$
10. $\frac{d}{dx}\sec x = \tan x \sec x$
11. $\frac{d}{dx}\csc x = -\cot x \csc x$
12. $\frac{d}{dx}e^x = e^x$
13. $\frac{d}{dx}\ln x = \frac{1}{x}$
14. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
15. $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$
16. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$

Integrals

1. $\int af(x)dx = a\int f(x)dx$
2. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
3. $\int x^m dx = \frac{x^{m+1}}{m+1} (m \neq -1)$
 $= \ln x (m = -1)$
4. $\int \sin x dx = -\cos x$
5. $\int \cos x dx = \sin x$
6. $\int \tan x dx = \ln|\sec x|$

$$7. \int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$8. \int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$9. \int \sin ax \cos ax \, dx = -\frac{\cos 2ax}{4a}$$

$$10. \int e^{ax} \, dx = \frac{1}{a}e^{ax}$$

$$11. \int xe^{ax} \, dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$12. \int \ln ax \, dx = x \ln ax - x$$

$$13. \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$14. \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right|$$

$$15. \int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a}$$

$$16. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

$$17. \int \sqrt{a^2 + x^2} \, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$$

$$18. \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$